Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

cut at this point is known. Designating the co-ordinates of $P$, just found, by $(x'', y'')$ and the co-ordinates of $C$ by $(x',y')[=(o, a)]$, $CP=\sqrt{(x''-x')^2+(y''-y')^2}$ ....(3).

Measure this distance, $CP$, and fix a stake at $P$.

The equation of $SA$ is $x=\frac{b}{2}$ ....(4).

Substituting this value of $x$ in (1), the cut at $S$ is at once obtained. Denoting co-ordinates of $S$ by $(x'',y'')$ and co-ordinates of $C$ by $(x',y')$, and substituting in (3), $CS$ is known. Measure this distance, and fix a stake at $S$.

The same course of reasoning applies in finding the cuts at $P'$ and $T$.

The writer has found this method more expeditious than *trial* and *error* when the surface was much inclined. The numerical computations did not require as much labor as maneuvering with the rod and level.

Since he has not seen the above method in any of the books which have fallen under his eyes, he has been induced to give it in the hope that it might prove useful, as well as suggestive, to others under similar circumstances.

## NOTE ON MR. ELLWOOD'S REMARKS ON DIVISION.

By **DAVID EUGENE SMITH, Ph. D.,** Professor of Mathematics in the Michigan State Normal School, Ypsilanti, Michigan.

Reluctantly, feeling that it is almost unnecessary, a note is offered on Mr. Ellwood's article on p. 47. Such articles float through the primary journals of education now and then, and possibly do no harm that can be undone by replying to them. But in a mathematical journal such pedagogy should not go unchallenged.

It is a trite idea that all advance in mathematics, and in all science generally, has had to contend with just such arguments as those of Mr. Ellwood. The whole history of the development of mathematics has been a history of the destruction of old definitions, old hobbies, old idols. When Ahmes copied the papyrus which now, after 3500 years, makes him famous, only one fraction could be written whose numerator was not one. A definition or an artificial symbolism created a barrier. For over a thousand years thereafter the Egyptians, Hebrews, Phœnicians, and Greeks recognized that barrier. But finally came an iconoclast and knocked down the sacred wall, and the Greeks learned about fractions.

The Arabs found negative roots to certain quadratic equations. Teachers raised their hands in horror. It was impossible; the sacred definition of number stood in the way. But by and by some one suggested that that definition better stand aside, and so the reality of negative numbers became recognized. Then came some other impossible roots, expressions that involved the square root of a negative number. Even the serious thinkers among the Arabs rejected them. Cardan in 1545,saw some light on the subject, but even from the mathematicians of his time the definitions of number and square-root barred such seemingly impossible roots. But as this century approached the light grew stronger and finally Gauss and Argand knocked down the barriers and the theory of complex numbers was established.

It was the same with the theory of exponents. $4^2$ meant 4 taken twice as a factor. Then some one had the audacity to suggest $4^{\frac{1}{2}}$, and we can fancy the cries that greeted it; and we can almost hear the definition-defenders talking about the absurdity of taking a number one-half of a time as a factor. How they must have writhed when $4^{-\frac{1}{2}}$was first suggested! It was the same old trouble. Somebody saw that a pet definition must be shattered, and amid tears and complaints the fragments were swept out.

It is within our generation that the idol known as the commutative law of multiplication has been placed in the museum of antiquities. One of two things must happen, quaternions must die in the womb, or else the commutative law must yield. It was too bad, but quaternions conquered. And now the theory of substitutions supports quaternions with such a powerful arm that even primary text-books are properly limiting the application of the law.

Of course the list could be multiplied a thousand fold. But the moral is that the world is going to move along in spite of somebody's definition of fraction, or number, or power, or exponent, or multiplication, or division, and in spite of our veneration for commutative laws, the word "imaginary," such ideas as the primative one of interior angles, and the multifold like.

But Mr. Ellwood will say, that this does not touch his article. On p. 47 he says, "Multiplication is a mere process of adding." Here is the idol, the sacred definition. He might as will have said, "One is not a number," because nobody ever thought of it as such until modern times. So the enlargement of the notion of multiplication is modern. But the enlargement is made, and it will never be unmade. Multiplication was, originally, what Mr. Ellwood says, but the world of to-day says, "*nous avons change tout cela.*" And the world of to-day says $\$10 \div 2 = \$5$, and a good part of the world expressly distinguishes between the two notions of division in the primary grades. To try to establish the limitation proposed by Mr. Ellwood is to let the tail try to wag the dog.